Signal and Systems

By

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كلية الهندسة بشبرا

FACULTY OF ENGINEERING- SHOUBRA

Lecture (3)

Course Title: **Signal and Systems**

Course Code: **ELE 115**

Contact Hours: **5.**

= [2 Lect. + 2 Tut + 1 Lab]

Assessment:

Final Exam: 75%.

Midterm: ??%.

Year Work & Quizzes: 50%.

Experimental/Oral: 25%.

Textbook:

1- E. W. Kamen and B. S. Heck, Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd ed., Pearson Hihgher Education, 2006.
2- Benjamin C. Kuo " Automatic control systems" 9th ed., John Wiley & Sons, Inc. 2010.
3- Katsuhiko Ogata, "Modern Control Engineering", 4th Edition, 2001.

Course Description

- Introduction, fundamentals and basic properties of signals and systems, definition of open loop and closed loop systems, mathematical models of physical systems (mechanical, electrical, electromechanical systems …), control system components, block diagram simplification, signal flow graph, state variable models, Z-Transform and its properties, solving difference equations, pulse transfer function of discrete system, Fourier transforms, continuous and discrete signal analysis, transient response of first and second order control systems, real life applications such as analog and digital filters, introduction to basics of digital signal processor (DSP) and its features and capabilities of commercial applications.

Mathematical modeling of linear dynamic systems & transfer function

Block Diagram Fundamentals

&

Reduction Techniques

Superposition of Multiple Inputs

Step 1: Set all inputs except one equal to zero.

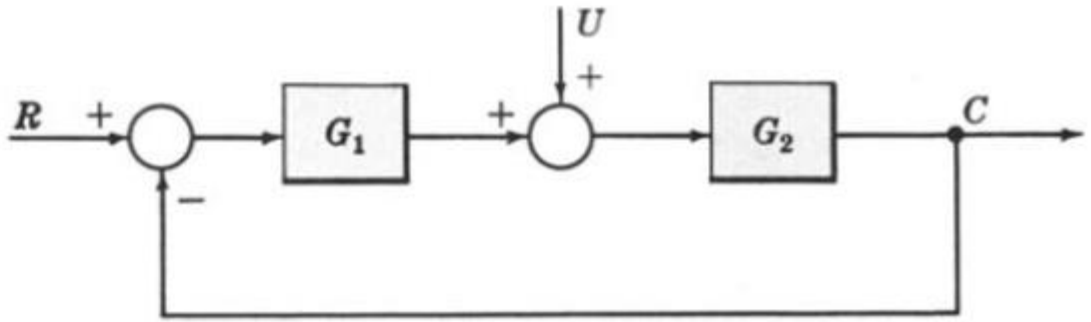
Step 2: Transform the block diagram to canonical form, using the transformations of Section 7.5.

Step 3: Calculate the response due to the chosen input acting alone.

Step 4: Repeat Steps 1 to 3 for each of the remaining inputs.

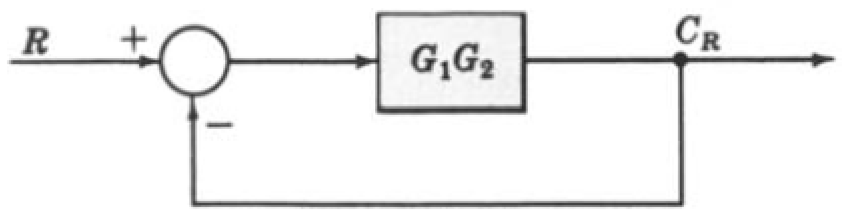
Step 5: Algebraically add all of the responses (outputs) determined in Steps 1 to 4. This sum is the total output of the system with all inputs acting simultaneously.

Example-12: Multiple Input System. Determine the output C due to inputs R and U using the Superposition Method.



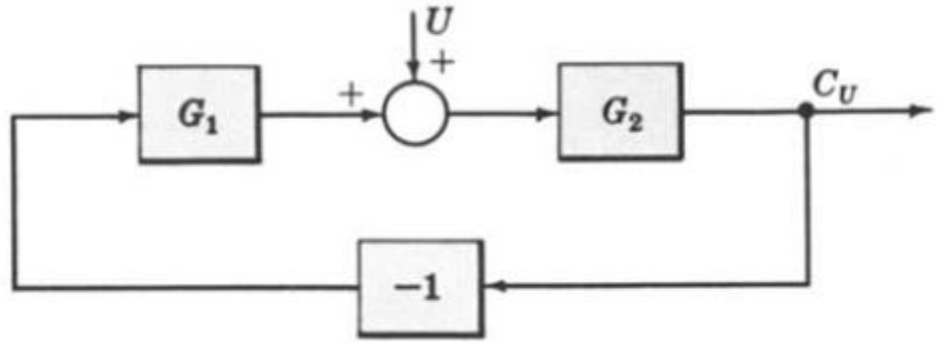
Step 1: Put $U \equiv 0$.

Step 2: The system reduces to



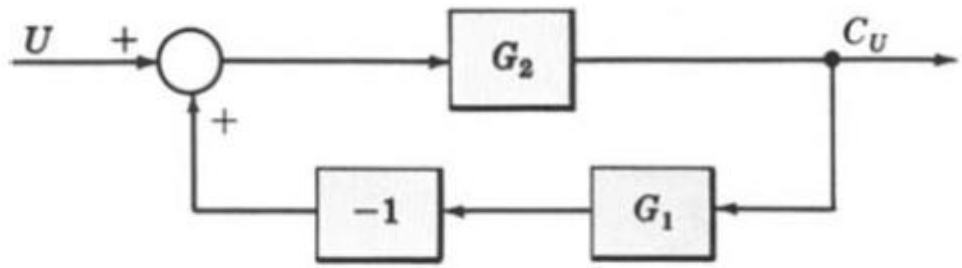
Step 3: the output C_R due to input R is $\boxed{C_R = [G_1G_2/(1 + G_1G_2)]R}$

Example-12: Continue.

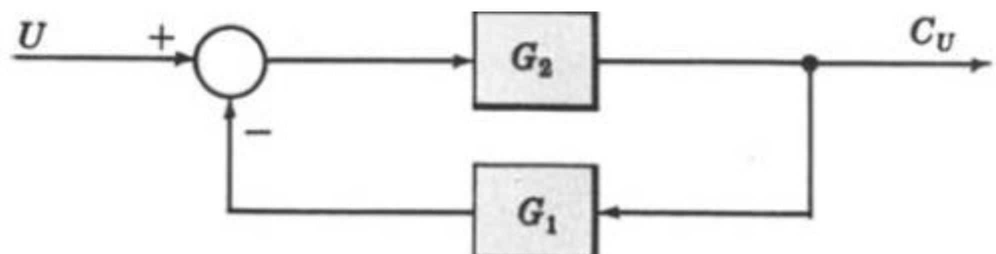


Step 4a: Put $R = 0$.
Step 4b: Put -1 into a block, representing the negative feedback effect:

Rearrange the block diagram:



Let the -1 block be absorbed into the summing point:



Step 4c: the output C_U due to input U is $\boxed{C_U = [G_2/(1 + G_1G_2)]U.}$

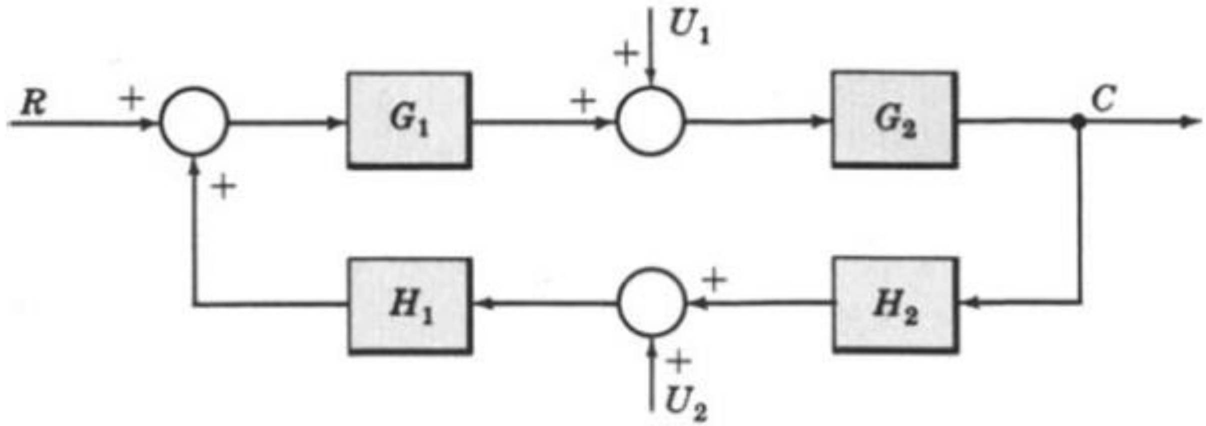
Example-12: Continue

Step 5: The total output is $C = C_R + C_U$

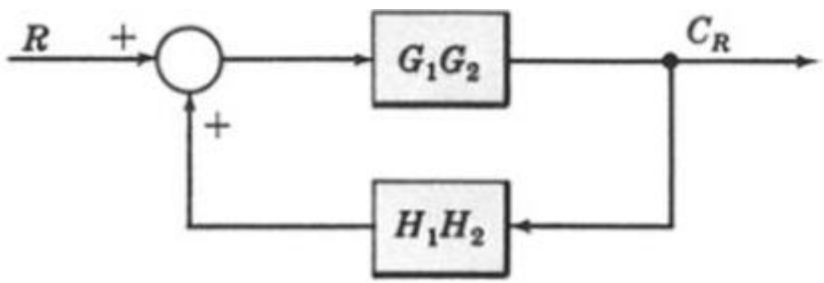
$$= \left[\frac{G_1 G_2}{1 + G_1 G_2} \right] R + \left[\frac{G_2}{1 + G_1 G_2} \right] U$$

$$= \left[\frac{G_2}{1 + G_1 G_2} \right] [G_1 R + U]$$

Example-13: Multiple-Input System. Determine the output C due to inputs R, U_1 and U_2 using the Superposition Method.



Let $U_1 = U_2 = 0$.

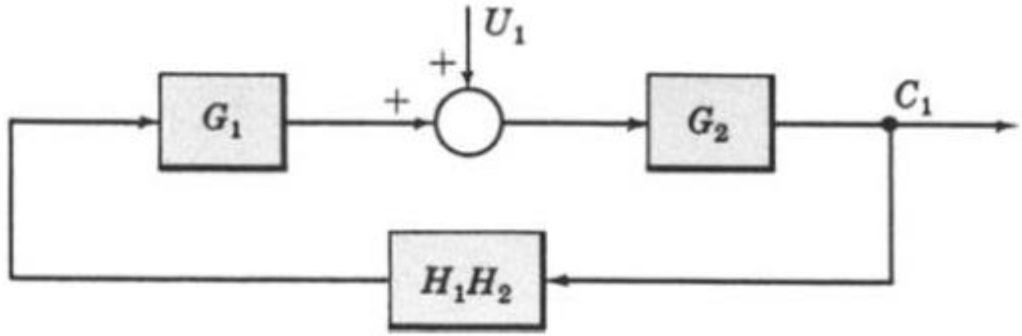


$$C_R = [G_1G_2/(1 - G_1G_2H_1H_2)]R$$

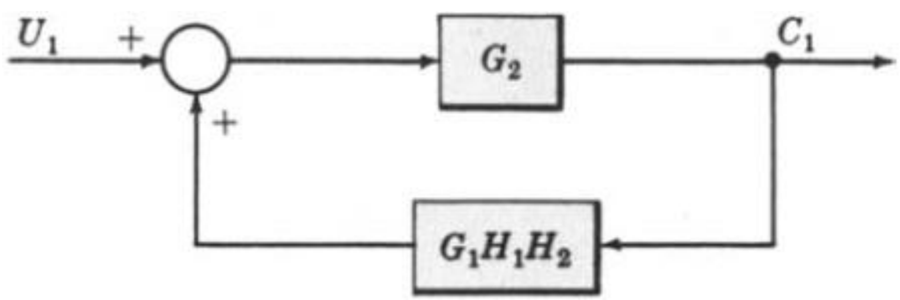
where C_R is the output due to R acting alone.

Example-13: Continue

Now let $R = U_2 = 0$.



Rearranging the blocks, we get

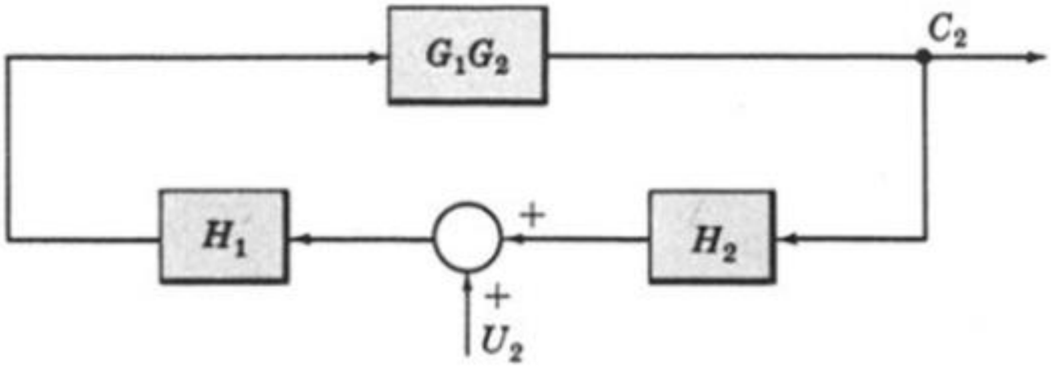


$$\boxed{C_1 = [G_2/(1 - G_1G_2H_1H_2)]U_1}$$

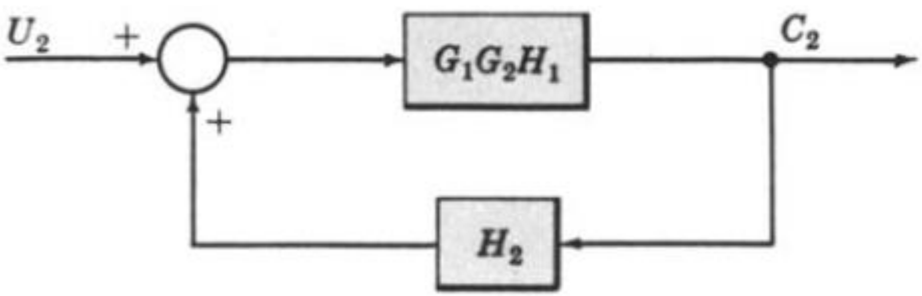
where C_1 is the response due to U_1 acting alone.

Example-13: Continue

Finally, let $R = U_1 = 0$.



Rearranging the blocks, we get



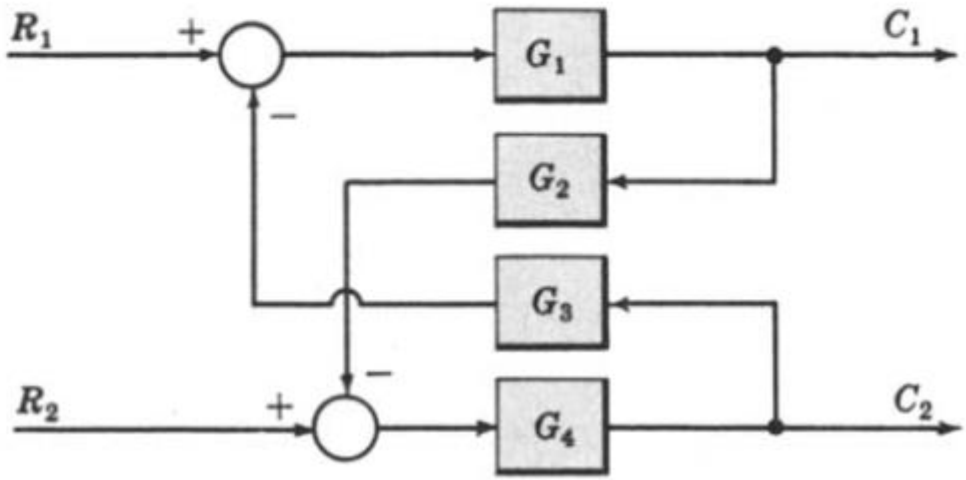
$$C_2 = [G_1G_2H_1/(1 - G_1G_2H_1H_2)]U_2$$

where C_2 is the response due to U_2 acting alone.

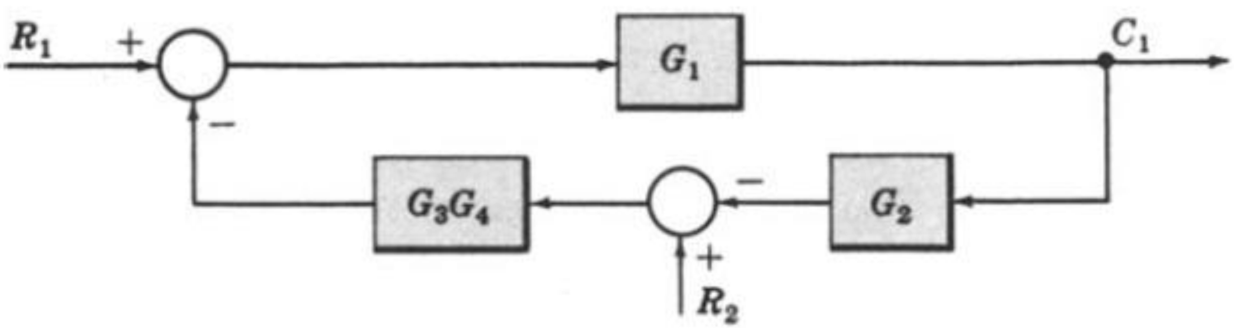
By superposition, the total output is

$$C = C_R + C_1 + C_2 = \frac{G_1G_2R + G_2U_1 + G_1G_2H_1U_2}{1 - G_1G_2H_1H_2}$$

Example-14: Multi-Input Multi-Output System. Determine C_1 and C_2 due to R_1 and R_2.

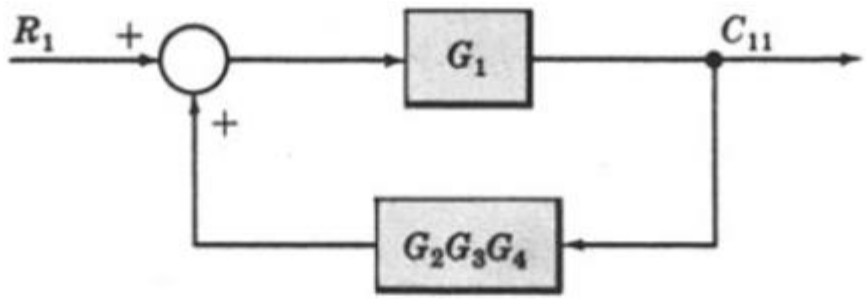


First ignoring the output C_2.



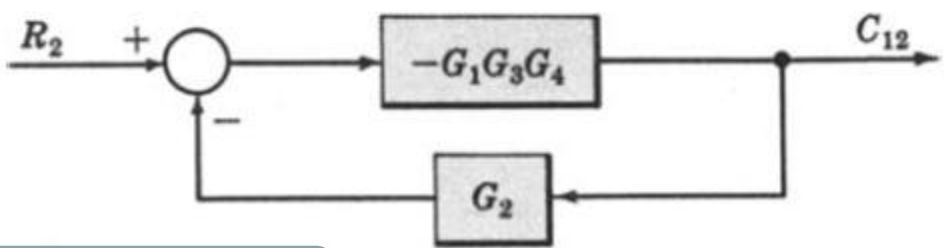
Example-14: Continue

Letting $R_2 = 0$ and combining the summing points,



Hence C_{11}, the output at C_1 due to R_1 alone, is $\boxed{C_{11} = G_1 R_1/(1 - G_1G_2G_3G_4).}$

For $R_1 = 0$,

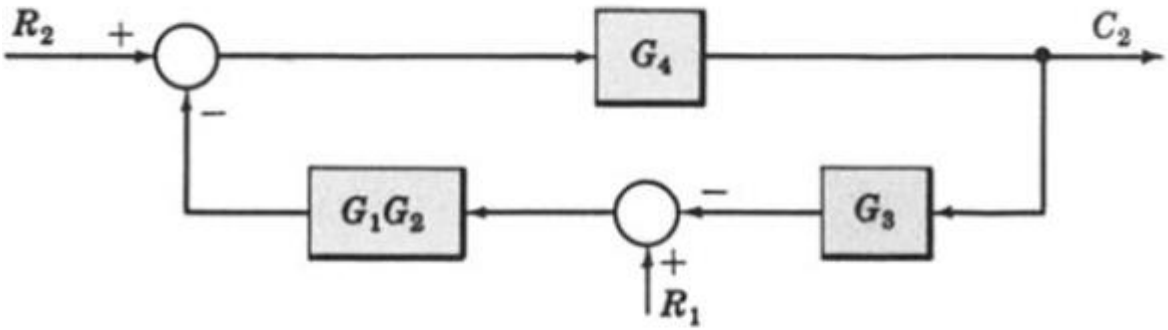


Hence $\boxed{C_{12} = -G_1G_3G_4R_2/(1 - G_1G_2G_3G_4)}$ is the output at C_1 due to R_2 alone.

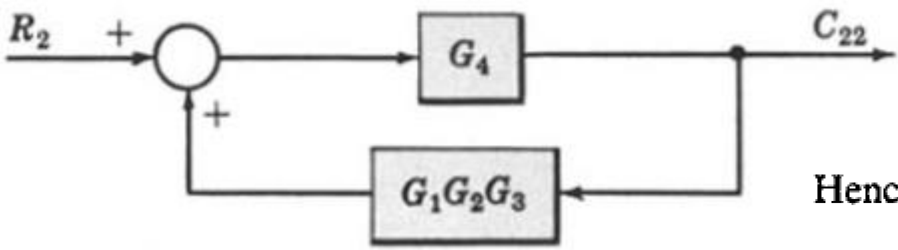
Thus $\boxed{C_1 = C_{11} + C_{12} = (G_1R_1 - G_1G_3G_4R_2)/(1 - G_1G_2G_3G_4)}$

Example-14: Continue

Now we reduce the original block diagram, ignoring output C_1.

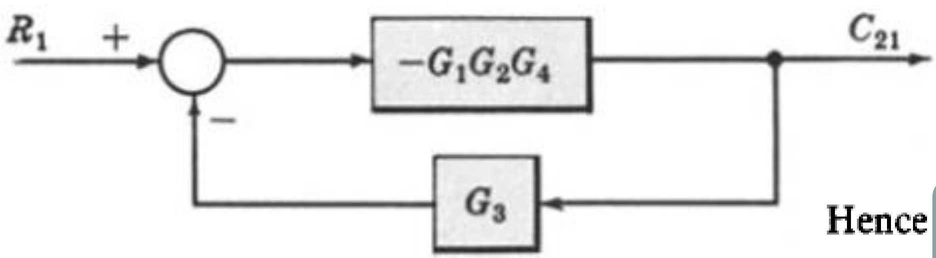


When R1 = 0,



Hence $\boxed{C_{22} = G_4R_2/(1 - G_1G_2G_3G_4)}$

When R2 = 0,

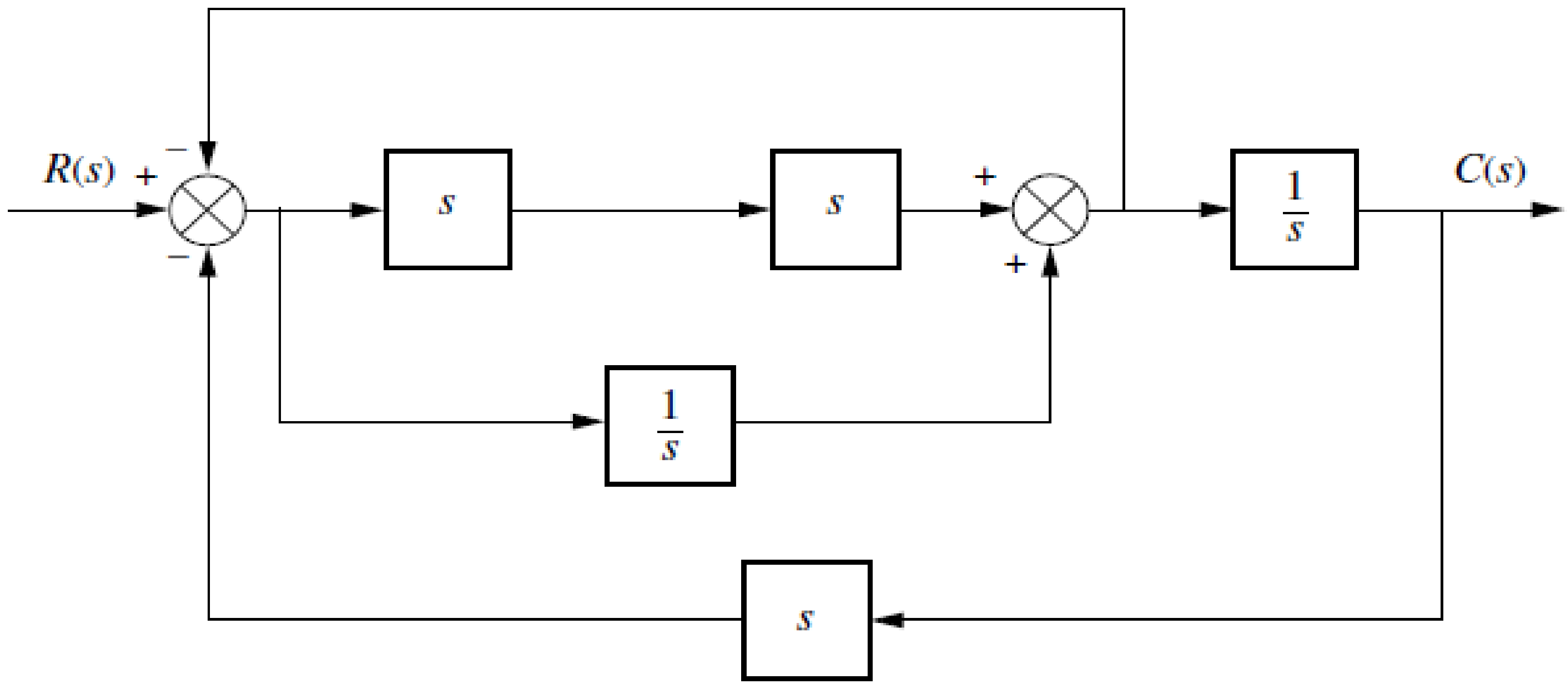


Hence $\boxed{C_{21} = -G_1G_2G_4R_1/(1 - G_1G_2G_3G_4)}$

Finally, $\boxed{C_2 = C_{22} + C_{21} = (G_4R_2 - G_1G_2G_4R_1)/(1 - G_1G_2G_3G_4)}$

Skill Assessment Exercise:

PROBLEM: Find the equivalent transfer function, $T(s) = C(s)/R(s)$, for the system

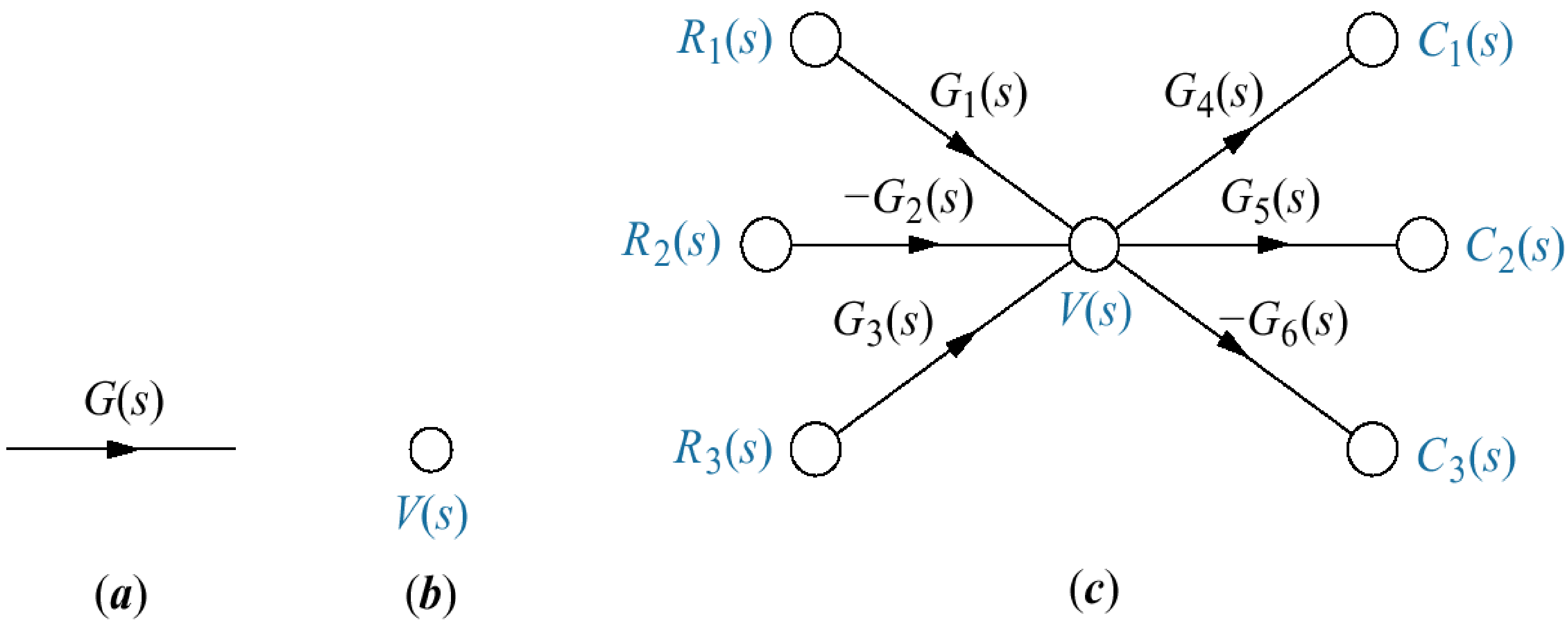


Answer of Skill Assessment Exercise:

ANSWER: $$T(s) = \frac{s^3 + 1}{2s^4 + s^2 + 2s}$$

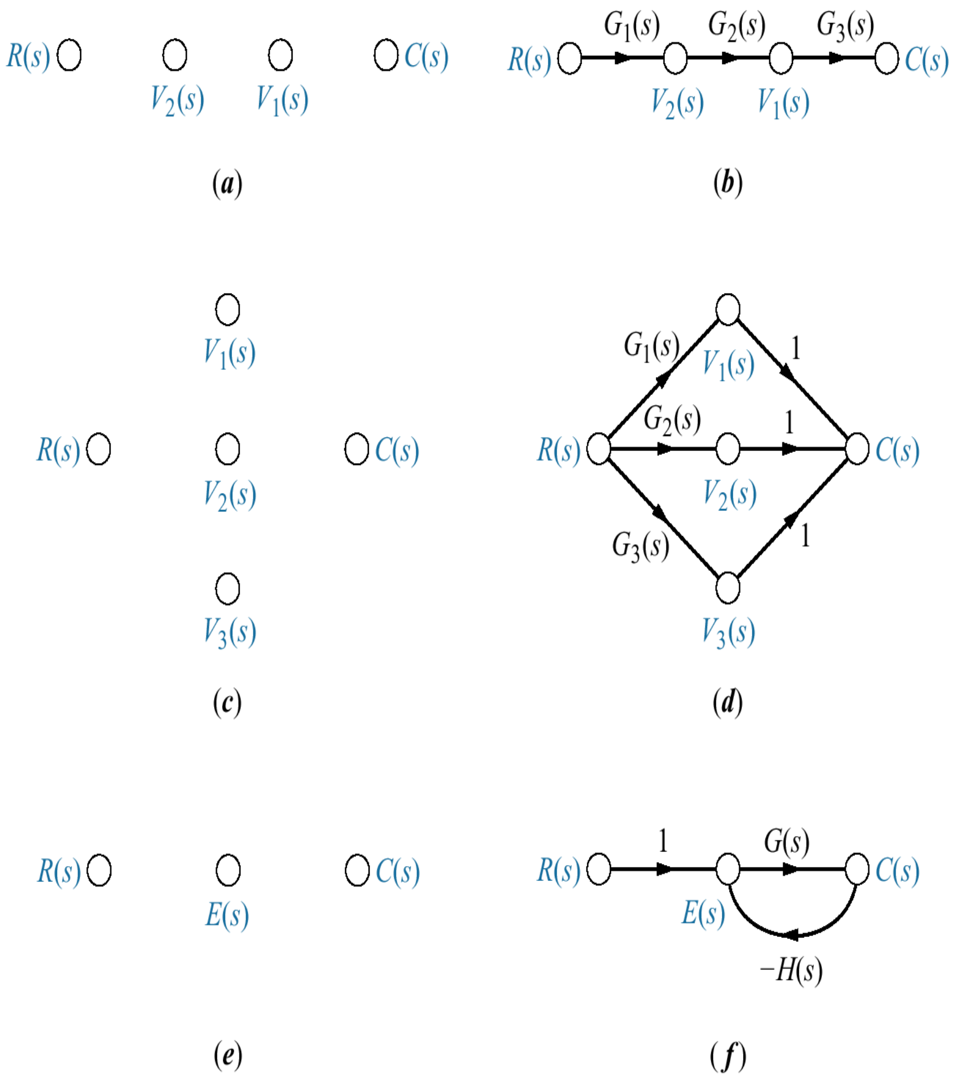
Signal-flow Graph Components

a. system;
b. signal;
c. interconnection of systems and signals



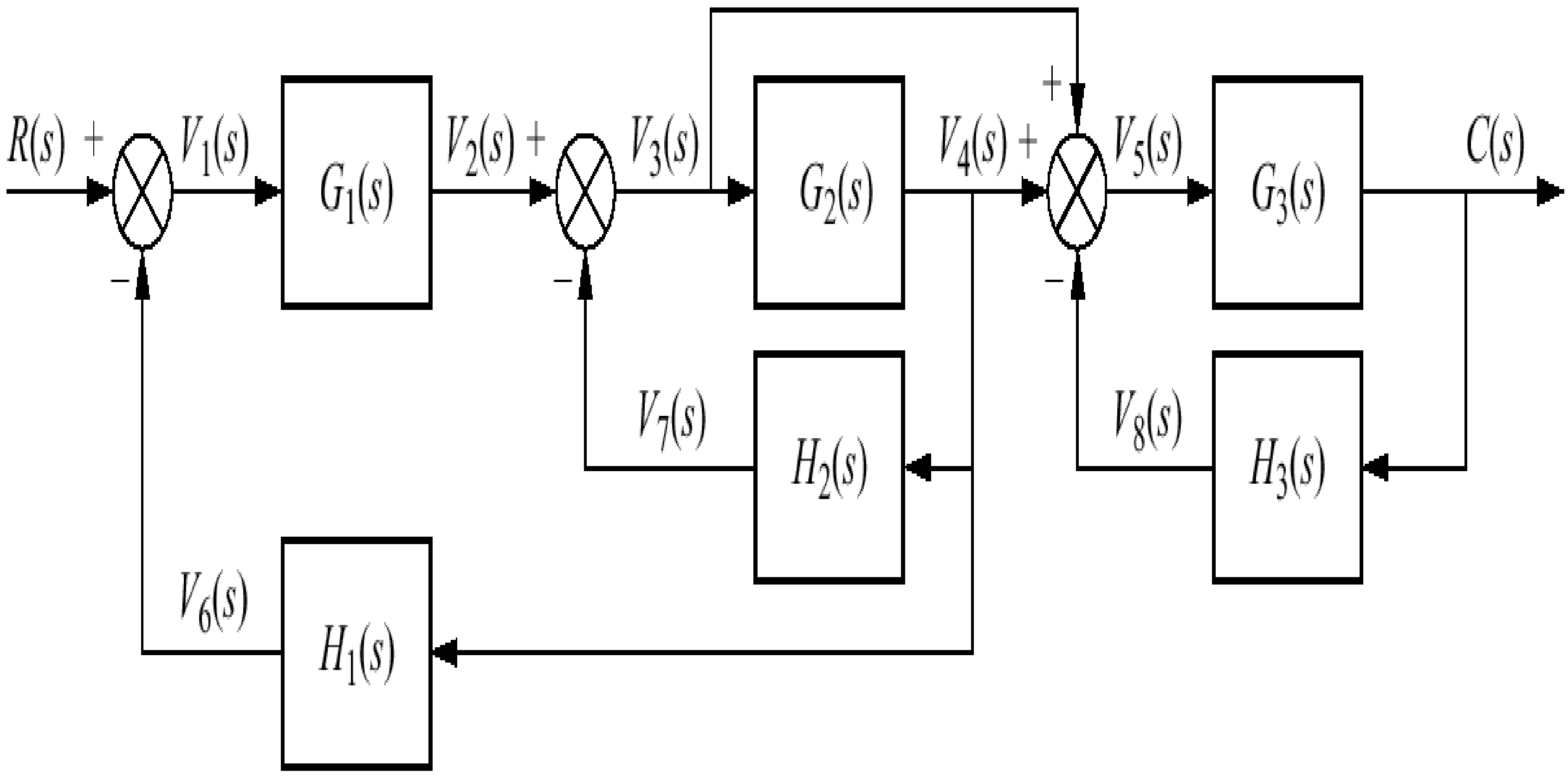
Building Signal-flow Graphs

a. Cascaded system nodes
b. Cascaded system signal-flow graph;
c. Parallel system nodes
d. Parallel system signal-flow graph;
e. Feedback system nodes
f. Feedback system signal-flow graph



Converting a Block Diagram to a Signal-flow Graph

Problem: Convert the block diagram to a signal-flow graph.



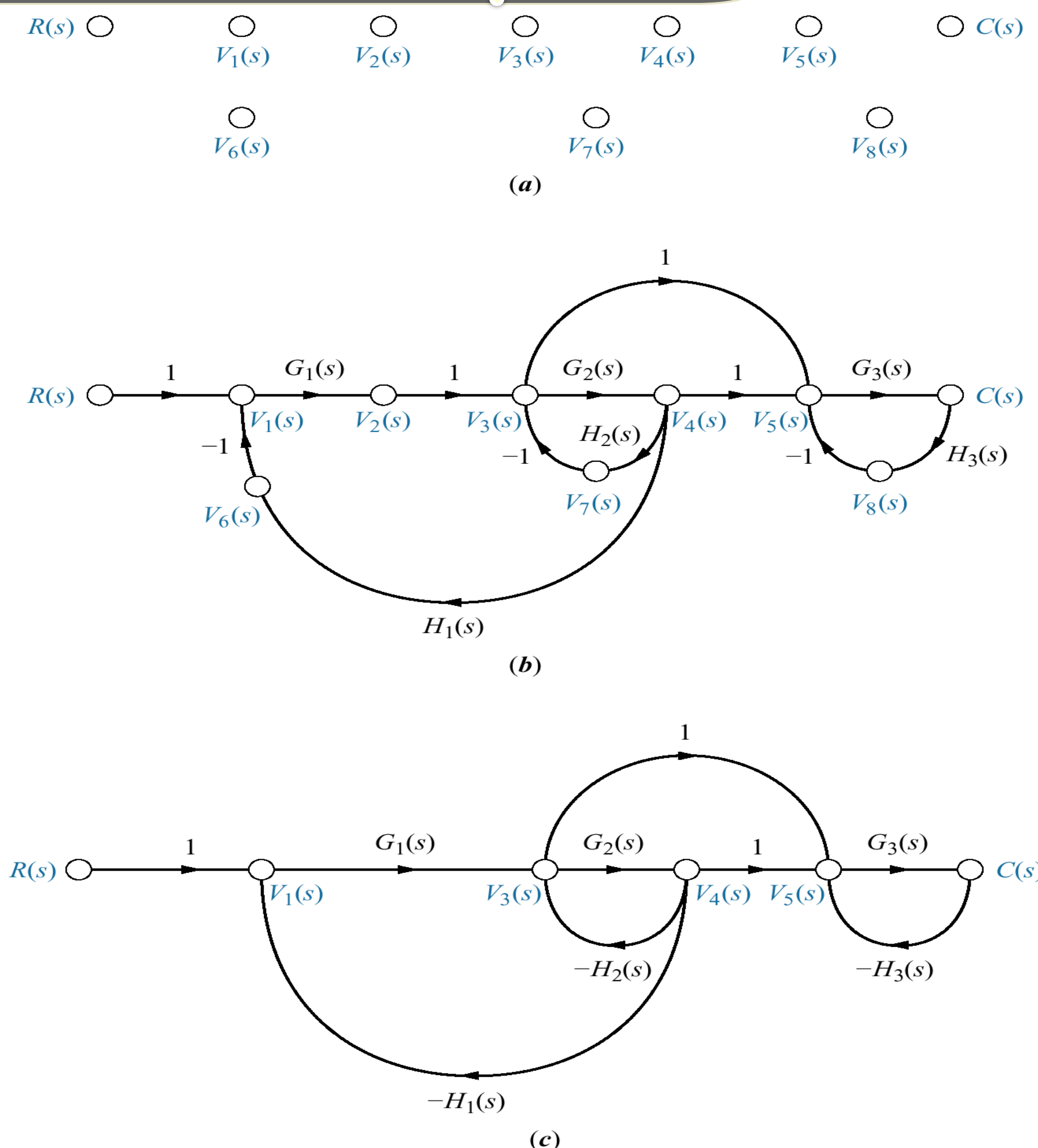
Converting a Block Diagram to a Signal-flow Graph

Signal-flow graph development:

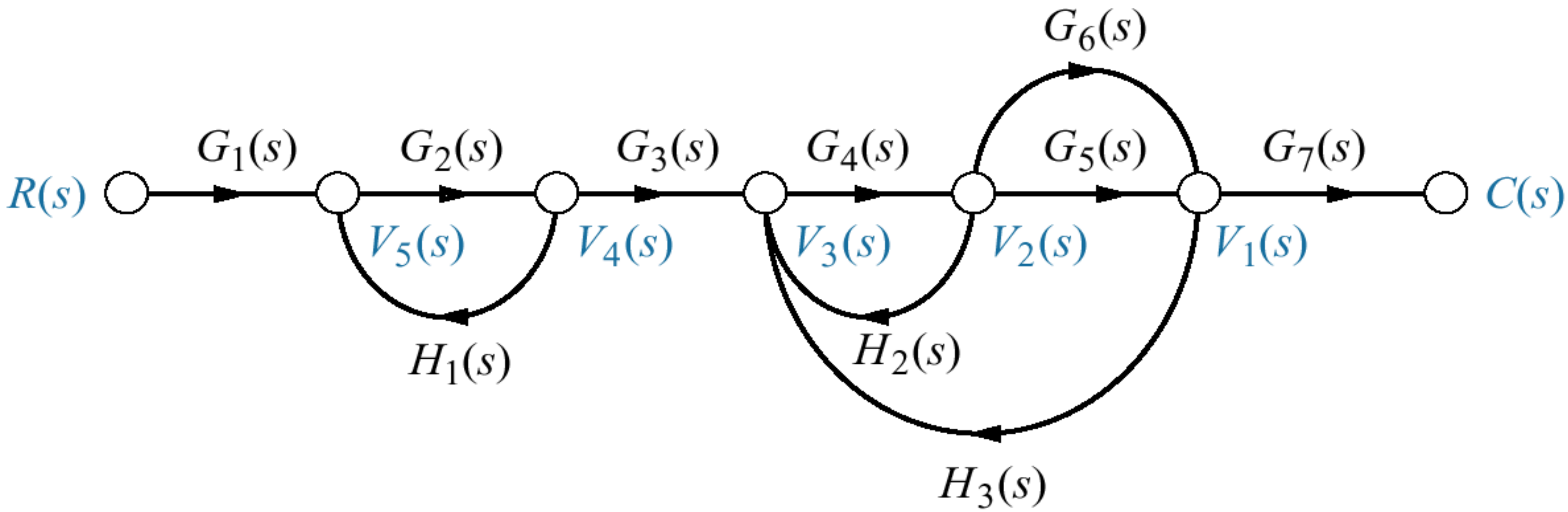
a. signal nodes;

b. signal-flow graph;

c. simplified signal-flow graph



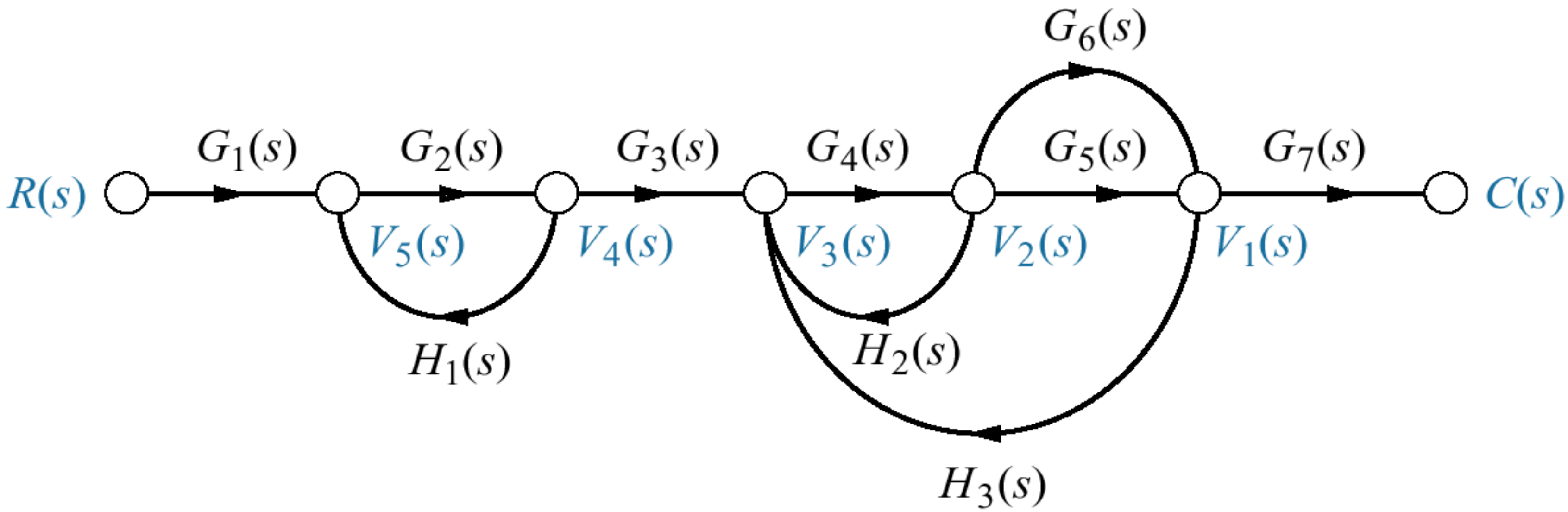
Mason's Rule - Definitions



Loop gain "L_k": The product of branch gains found by traversing a path that starts at a node and ends at the same node, following the direction of the signal flow, without passing through any other node more than once. $G_2(s)H_1(s)$, $G_4(s)H_2(s)$, $G_4(s)G_5(s)H_3(s)$, $G_4(s)G_6(s)H_3(s)$

Forward-path gain "P_k": The product of gains found by traversing a path from input node to output node in the direction of signal flow. $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$, $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$

Mason's Rule - Definitions



Nontouching loops: loops that do not have any nodes in common. $G_2(s)H_1(s)$ does not touch $G_4(s)H_2(s)$, $G_4(s)G_5(s)H_3(s)$, and $G_4(s)G_6(s)H_3(s)$

Nontouching-loop gain: The product of loop gains from nontouching loops taken 2, 3,4, or more at a time.

$[G_2(s)H_1(s)][G_4(s)H_2(s)]$, $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$, $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$

Mason's Rule

The Transfer function. C(s)/ R(s), of a system represented by a signal-flow graph is

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k P_k \Delta_k}{\Delta}$$

Where

K = number of forward paths

P_k = the k^{th} forward-path gain

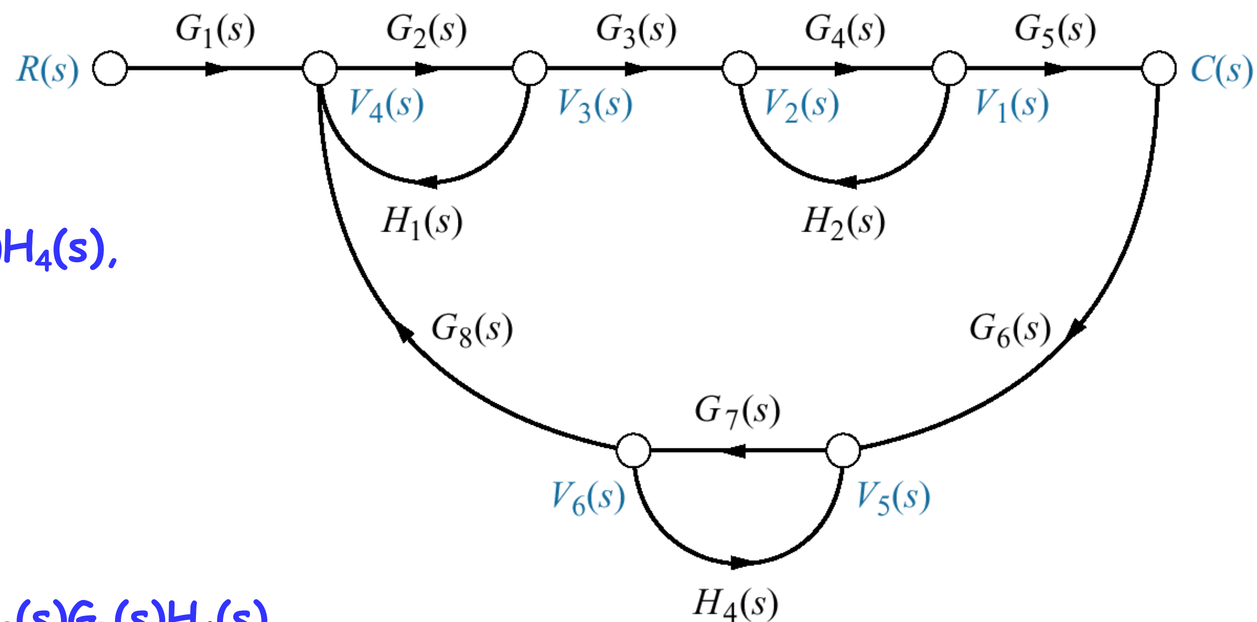
Δ = 1 - $\sum$loop gains + $\sum$ nontouching-loop gains taken 2 at a time - $\sum$ nontouching-loop gains taken 3 at a time + $\sum$ nontouching-loop gains taken 4 at a time - …….

Δ_k = 1 - loop gain terms that does not touch the kth forward path.

Transfer Function via Mason's Rule

Problem: Find the transfer function for the signal flow graph

Solution:



forward path

$G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)$

Loop gains

$G_2(s)H_1(s)$, $G_4(s)H_2(s)$, $G_7(s)H_4(s)$,
$G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)G_8(s)$

Nontouching loops

2 at a time

$G_2(s)H_1(s)G_4(s)H_2(s)$

$G_2(s)H_1(s)G_7(s)H_4(s)$

$G_4(s)H_2(s)G_7(s)H_4(s)$

3 at a time $G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)$

Now

$$\Delta = 1-[G_2(s)H_1(s)+G_4(s)H_2(s)+G_7(s)H_4(s)+ G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)G_8(s)] + [G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s) + G_4(s)H_2(s)G_7(s)H_4(s)] - [G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]$$

$$\Delta_1 = 1 - G7(s)H4(s)$$

$$G(s) = \frac{P_1\Delta_1}{\Delta} = \frac{[G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)][1-G_7(s)H_4(s)]}{\Delta}$$

With Our Best Wishes
Signals and Systems
Course Staff